

ON THE FOUNDATIONS OF THE TWO MEASURES FIELD THEORY

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Abstract

Two Measures Field Theory (TMT) uses both the Riemannian volume element $\sqrt{-g}d^4x$ and a new one Φd^4x where the new measure of integration Φ can be build of four scalar fields. Arguments in favor of TMT, both from the point of view of first principles and from the TMT results are summarized. Possible origin of the TMT and symmetries that protect the structure of TMT are reviewed. It appears that four measure scalar fields treated as "physical coordinates" allow to define local observables in quantum gravity. The resolution of the old cosmological constant problem as a possible direct consequence of the TMT structure is discussed. Other applications of TMT to cosmology and particle physics are also mentioned.

I. MAIN IDEAS OF THE TWO MEASURES FIELD THEORY

TMT is a generally coordinate invariant theory where *all the difference from the standard field theory in curved space-time consists only of the following three additional assumptions:*

1. The first assumption is the hypothesis that the effective action at the energies below the Planck scale has to be of the form [1]- [11]

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$$S = \int L_1 \Phi d^4x + \int L_2 \sqrt{-g} d^4x \quad (1)$$

including two Lagrangians L_1 and L_2 and two measures of integration $\sqrt{-g}$ and Φ or, equivalently, two volume elements Φd^4x and $\sqrt{-g} d^4x$ respectively. One is the usual measure of integration $\sqrt{-g}$ in the 4-dimensional space-time manifold equipped with the metric $g_{\mu\nu}$. Another is the new measure of integration Φ in the same 4-dimensional space-time manifold. The measure Φ being a scalar density and a total derivative may be defined¹

- either by means of four scalar fields φ_a ($a = 1, 2, 3, 4$)

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d. \quad (2)$$

- or by means of a totally antisymmetric three index field $A_{\alpha\beta\gamma}$

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} \partial_\mu A_{\nu\alpha\beta}. \quad (3)$$

To provide parity conservation in the case given by Eq.(2) one can choose for example one of φ_a 's to be a pseudoscalar; in the case given by Eq.(3) we must choose $A_{\alpha\beta\gamma}$ to have negative parity. A special case of the structure (1) with definition (3) has been recently discussed in Ref. [13] in applications to supergravity and the cosmological constant problem.

2. It is assumed that the Lagrangians L_1 and L_2 are functions of all matter fields, the metric, the connection (or spin-connection) but not of the "measure fields" (φ_a or $A_{\alpha\beta\gamma}$).

3. Important feature of TMT that is responsible for many interesting and desirable results of the field theory models studied so far [1]- [10] consists of the assumption that all fields, including also metric, connection (or vierbein and spin-connection) and the *measure fields* (φ_a or $A_{\alpha\beta\gamma}$) are independent dynamical variables. All the relations between them are results of equations of motion. In particular, the independence of the metric and the connection means that we proceed in the first order formalism and the relation between connection and metric is not necessarily according to Riemannian geometry.

¹For applications of the measure Φ in string and brane theories see Ref. [12].

We want to stress again that except for the listed three assumptions in our TMT models we do not make any changes as compared with principles of the standard field theory in curved space-time. In other words, all the freedom in constructing different models in the framework of TMT consists of the choice of the concrete matter content and the Lagrangians L_1 and L_2 that is quite similar to the standard field theory.

Since Φ is a total derivative, a shift of L_1 by a constant, $L_1 \rightarrow L_1 + const$, has no effect on the equations of motion. Similar shift of L_2 would lead to the change of the constant part of the Lagrangian coupled to the volume element $\sqrt{-g}d^4x$. In the standard GR, this constant term is the cosmological constant. However in TMT the relation between the constant term of L_2 and the physical cosmological constant is very non trivial (see [3]- [5]).

In the case of the definition of Φ by means of Eq.(2), varying the measure fields φ_a , we get

$$B_a^\mu \partial_\mu L_1 = 0 \quad \text{where} \quad B_a^\mu = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d. \quad (4)$$

Since $Det(B_a^\mu) = \frac{4^{-4}}{4!} \Phi^3$ it follows that if $\Phi \neq 0$,

$$L_1 = sM^4 = const \quad (5)$$

where $s = \pm 1$ and M is a constant of integration with the dimension of mass.

In the case of the definition (3), variation of $A_{\alpha\beta\gamma}$ yields

$$\varepsilon^{\mu\nu\alpha\beta} \partial_\mu L_1 = 0, \quad (6)$$

that implies Eq.(5) without the condition $\Phi \neq 0$ needed in the model with four scalar fields φ_a .

One should notice *the very important differences of TMT from scalar-tensor theories with nonminimal coupling*:

a) In general, the Lagrangian density L_1 (coupled to the measure Φ) may contain not only the scalar curvature term (or more general gravity term) but also all possible matter fields terms. This means that TMT modifies in general both the gravitational sector and the

matter sector; b) If the field Φ were the fundamental (non composite) one then instead of (5), the variation of Φ would result in the equation $L_1 = 0$ and therefore the dimensionfull integration constant M^4 would not appear in the theory.

Applying the Palatini formalism in TMT one can show (see for example [3], [11]) that the resulting relation between metric and connection includes also the gradient of the ratio of the two measures

$$\zeta \equiv \frac{\Phi}{\sqrt{-g}} \quad (7)$$

which is a scalar field. The gravity and matter field equations obtained by means of the first order formalism contain both ζ and its gradient. It turns out that at least at the classical level, the measure fields affect the theory only through the scalar field ζ .

The consistency condition of equations of motion has the form of a constraint which determines $\zeta(x)$ as a function of matter fields. The surprising feature of the theory is that neither Newton constant nor curvature appear in this constraint which means that the *geometrical scalar field $\zeta(x)$ is determined by the matter fields configuration* locally and straightforward (that is without gravitational interaction).

By an appropriate change of the dynamical variables which includes a conformal transformation of the metric, one can formulate the theory in a Riemannian (or Riemann-Cartan) space-time. The corresponding conformal frame we call "the Einstein frame". The big advantage of TMT is that in the very wide class of models, *the gravity and all matter fields equations of motion take canonical GR form in the Einstein frame*. All the novelty of TMT in the Einstein frame as compared with the standard GR is revealed only in an unusual structure of the scalar fields effective potential (produced in the Einstein frame), masses of fermions and their interactions with scalar fields as well as in the unusual structure of fermion contributions to the energy-momentum tensor: all these quantities appear to be ζ dependent. This is why the scalar field $\zeta(x)$ determined by the constraint as a function of matter fields, has a key role in dynamics of TMT models.

II. POSSIBLE ORIGIN OF TMT FROM LOW ENERGY LIMIT OF THE BRANE-WORLD SCENARIO

As for the possible origin of the modified measure one can think of the following arguments. Let us start by noticing that the modified measure part of the action (1), $\int L_1 \Phi d^4x$, in the case $L_1 = \text{const}$ becomes a topological contribution to the action, since Φ is a total derivative. It is very interesting that this structure can be obtained by considering the topological theory which results from studying "space-time filling branes", Refs. [14]- [16].

Following for example Ref. [15], the Nambu-Goto action of a 3-brane embedded in a 4-dimensional space-time²

$$S_{NG} = T \int d^4x \sqrt{|\det(\partial_\mu \varphi^a \partial_\nu \varphi^b G_{ab}(\varphi))|} \quad (8)$$

(where $G_{ab}(\varphi)$ is the metric of the embedding space), is reduced to

$$\frac{T}{4!} \int d^4x \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d \sqrt{|\det G_{ab}(\varphi^a)|} = \frac{T}{4!} \int \Phi \sqrt{|\det G_{ab}(\varphi^a)|} d^4x. \quad (9)$$

where Φ is determined by Eq.(2). The Euler-Lagrange equations for φ^a are identities which is the result of the fact [15] that the action (9) is topological.

This is the action of a "pure" space-filling brane governed just by the identically constant brane tension. Let us now assume that the 3-brane is equipped with its own metric $g_{\mu\nu}$ and connection. If we want to describe a space-filling brane where gravity and matter are included by means of the Lagrangian $L_1 = L_1(g_{\mu\nu}, \text{connection}, \text{matter fields})$, we are led to the following form of the brane contribution to the action

$$S_1 = \int L_1 \Phi \sqrt{|\det G_{ab}(\varphi^a)|} d^4x \quad (10)$$

²It should be pointed out that string and brane theories can be formulated with the use of a modified measure [12] giving some new interesting results. In the spirit of the interpretation developed in this section, such strings or branes could be regarded as extended objects moving in an embedding extended object of the same dimensionality.

Recall that $\sqrt{|\det G_{ab}(\varphi^a)|}$ is a brane scalar.

If L_1 is not identically a constant (we have seen above that it may become a constant on the "mass shell", i.e. when equations of motion are satisfied), then we are not talking any more of a topological contribution to the action. Assuming again that L_1 is φ^a independent, it is easy to check that in spite of emergence of an additional factor $\sqrt{|\det G_{ab}(\varphi^a)|}$ in Eq.(10) as compared with the first term in Eq.(1), variation of φ^a yields exactly the same equation (5) if $\Phi \neq 0$. Moreover, all other equations of motion of the two measures theory remains unchanged. The only effect of this additional factor consists in the redefinition of the scalar field ζ , Eq.(7), where $\sqrt{|\det G_{ab}(\varphi^a)|}$ emerges as an additional factor. Notice that the same results are obtained if instead of $\sqrt{|\det G_{ab}(\varphi^a)|}$ in Eq.(10) there will be arbitrary function of φ^a .

Continuing discussion of the general structure of the action in TMT one of course may ask: why 3-brane moving only in $3 + 1$ dimensional embedding space-time? This can be obtained also starting from higher dimensional "brane-world scenarios". Indeed, let us consider for example a 3-brane evolving in an embedding 5-dimensional space-time with

$$ds^2 = G_{AB}dx^A dx^B = f(y)\hat{g}_{\mu\nu}(x^\alpha)dx^\mu dx^\nu + \gamma^2(y)dy^2, \quad A, B = 0, 1, \dots, 4; \quad \mu, \nu = 0, 1, 2, 3. \quad (11)$$

Assuming that it is possible to ignore the motion of the brane in the extra dimension, i.e. studying the brane with a fixed position in extra dimension, $y = \text{const}$, one can repeat the above arguments (starting with the Nambu-Goto action) where one needs to use only four functions $\varphi^a(x^\mu)$ which together with the fifth (x^μ independent) component along the axis y constitute the 5-vector describing the embedding of our brane; L_1 is again the φ^a independent Lagrangian of the gravity³ and matter on the brane. As a result we obtain exactly the same effective action as in Eq.(10) which describes a brane moving in the hypersurface $y = \text{const}$

³Notice that connection coefficients of our 3-brane are those where indexes run only from zero to three and these components do not suffer from discontinuities across the brane [17]

of the 5-dimensional space-time.

The brane theory action contains also a piece coming from the bulk dynamics. We will assume that gravity and matter exist also in the bulk where their action can be written in the form

$$S_{bulk} = \int \sqrt{-\hat{g}} f^2(y) \gamma(y) L_{bulk} d^4x dy \quad (12)$$

where $\hat{g} \equiv \det(\hat{g}_{\mu\nu})$ and L_{bulk} is the Lagrangian of the gravity and matter in the bulk. We are not interested in the dynamics in the hole bulk but rather in the effect of the bulk on the 4-dimensional dynamics. For this purpose one can integrate out the perpendicular coordinate y in the action (12). One can think of this integration in a spirit of the procedure known as averaging (for the case of compact extra dimensions see for example Ref. [18]). We do not perform this integration explicitly⁴ here but we expect that the resulting averaged contribution of the bulk dynamics to the 4-dimensional action one can write down in the form

$$S_2 = \int \sqrt{-\hat{g}} L_2(\hat{g}_{\mu\nu}, connection, matter fields) d^4x \quad (13)$$

that we would like to use as the second term of the postulated in Eq.(1) general form of the TMT action in four dimensions.

Notice however that the geometrical objects in these two actions may not be identical. The simplest assumption would be of course to take $\hat{g}_{\mu\nu} \equiv g_{\mu\nu}$ (and also coinciding connections). One may allow for the case $\hat{g}_{\mu\nu} \neq g_{\mu\nu}$ nevertheless. In fact, brane theory allows naturally bimetric theories [19] even if one starts with a single bulk metric. In our case this can be due to the fact that the metric at the brane and its average value may be different. The bimetric theories [20] give in any case only one massless linear combination of the two metrics which one can identify as long distance gravity. Connections at the brane and its average value may be different as well. But we expect this difference to be small due to the

⁴The correspondent calculations must take into account the discontinuity constraints.

continuity of the relevant connection coefficients (see footnote 3). Therefore performing the integration over extra dimension y we are left with just one independent connection which is important for TMT where the first order formalism is supposed to be one of the basic principles.

III. SYMMETRIES

A. Volume preserving diffeomorphisms

The volume element Φd^4x is invariant under the area preserving diffeomorphisms [1], i. e. internal transformations in the φ_a space

$$\varphi_a \rightarrow \varphi'_a = \varphi'_a(\varphi_b), \quad a, b = 1, 2, 3, 4 \quad (14)$$

which satisfy the area preserving condition

$$\epsilon_{abcd} \frac{\partial \varphi'_a}{\partial \varphi_k} \frac{\partial \varphi'_b}{\partial \varphi_l} \frac{\partial \varphi'_c}{\partial \varphi_m} \frac{\partial \varphi'_d}{\partial \varphi_n} = \epsilon_{klmn}. \quad (15)$$

B. Symmetries related to the general structure of TMT

In such a case, i.e. when the measure fields enter in the theory only via the measure Φ , the action (1) respects (up to an integral of a total divergence) the infinite dimensional group of shifts [3] of the measure fields φ_a . In the case given by Eq.(2) these symmetry transformations have the form

$$\varphi_a \rightarrow \varphi_a + f_a(L_1), \quad (16)$$

where $f_a(L_1)$ are arbitrary functions of L_1 (see details in Ref. [3]); in the case given by Eq.(3) they read $A_{\alpha\beta\gamma} \rightarrow A_{\alpha\beta\gamma} + \varepsilon_{\mu\alpha\beta\gamma} f^\mu(L_1)$ where $f^\mu(L_1)$ are four arbitrary functions of L_1 and $\varepsilon_{\mu\alpha\beta\gamma}$ is numerically the same as $\varepsilon^{\mu\alpha\beta\gamma}$. One can hope that this symmetry should prevent emergence of a measure fields dependence in L_1 and L_2 after quantum effects are taken into account.

IV. SPONTANEOUSLY BROKEN GLOBAL SCALE INVARIANCE WITHOUT GOLDSTONE BOSON

Let us consider a model where the action (1) is invariant under symmetry transformations in such a way that the measure fields φ_a (or $A_{\alpha\beta\gamma}$) participate in the transformations as well. The matter content of our model includes the scalar field ϕ , two fermion fields (neutrino N and electron E) and electromagnetic field A_μ . We allow in both L_1 and L_2 all the usual terms considered in standard field theory models in curved space-time. To give an example, we present here a model [10] where keeping the general structure of Eq.(1) it is convenient to represent the action in the following form:

$$\begin{aligned}
S = & \int d^4x e^{\alpha\phi/M_p} (\Phi + b\sqrt{-g}) \left[-\frac{1}{\kappa} R(\omega, e) + \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} \right] - \int d^4x e^{2\alpha\phi/M_p} [\Phi V_1 + \sqrt{-g} V_2] \\
& - \int d^4x \sqrt{-g} \frac{1}{4} g^{\alpha\beta} g^{\mu\nu} F_{\alpha\mu} F_{\beta\nu} + \int d^4x e^{\alpha\phi/M_p} (\Phi + k\sqrt{-g}) \frac{i}{2} \sum_i \bar{\Psi}_i \left(\gamma^a e_a^\mu \vec{\nabla}_\mu^{(i)} - \vec{\nabla}_\mu^{(i)} \gamma^a e_a^\mu \right) \Psi_i \\
& - \int d^4x e^{\frac{3}{2}\alpha\phi/M_p} \left[(\Phi + h_E \sqrt{-g}) \mu_E \bar{E} E + (\Phi + h_N \sqrt{-g}) \mu_N \bar{N} N \right]. \tag{17}
\end{aligned}$$

Here Ψ_i ($i = N, E$) is the general notation for the fermion fields N and E ; $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$; μ_N and μ_E are the mass parameters; $\vec{\nabla}_\mu^{(N)} = \vec{\partial} + \frac{1}{2} \omega_\mu^{cd} \sigma_{cd}$, $\vec{\nabla}_\mu^{(E)} = \vec{\partial} + \frac{1}{2} \omega_\mu^{cd} \sigma_{cd} + ie A_\mu$; $R(\omega, V) = e^{a\mu} e^{b\nu} R_{\mu\nu ab}(\omega)$ is the scalar curvature; e_a^μ and ω_μ^{ab} are the vierbein and spin-connection; $g^{\mu\nu} = e_a^\mu e_b^\nu \eta^{ab}$ and $R_{\mu\nu ab}(\omega) = \partial_{\mu a} \omega_{\nu b} + \omega_{\mu a}^c \omega_{\nu cb} - (\mu \leftrightarrow \nu)$. V_1 and V_2 are constants with the dimensionality $(mass)^4$. When Higgs field is included into the model then V_1 and V_2 turn into functions of the Higgs field. As we will see later, in the Einstein frame V_1 , V_2 and $e^{\alpha\phi/M_p}$ enter in the effective potential of the scalar sector. Constants b, k, h_N, h_E are non specified dimensionless real parameters; α is a real parameter which we take to be positive.

The action (17) is invariant under the global scale transformations:

$$\begin{aligned}
e_\mu^a & \rightarrow e^{\theta/2} e_\mu^a, \quad \omega_{ab}^\mu \rightarrow \omega_{ab}^\mu, \quad \varphi_a \rightarrow \lambda_{ab} \varphi_b, \quad A_\alpha \rightarrow A_\alpha, \quad \phi \rightarrow \phi - \frac{M_p}{\alpha} \theta \\
\Psi_i & \rightarrow e^{-\theta/4} \Psi_i, \quad \bar{\Psi}_i \rightarrow e^{-\theta/4} \bar{\Psi}_i, \\
\text{where } \theta & = const, \quad \lambda_{ab} = const \quad \text{and} \quad \det(\lambda_{ab}) = e^{2\theta}, \tag{18}
\end{aligned}$$

when the measure fields φ_a are used for definition of the measure Φ , as in Sec.3.1. If the definition (3) of Sec.3.2 is used then the scale transformation of the totally antisymmetric three index potential $A_{\alpha\beta\gamma}$ should be: $A_{\alpha\beta\gamma} \rightarrow e^{2\theta} A_{\alpha\beta\gamma}$.

Variation of the measure fields φ_a (in the model with the definition (2) or $A_{\alpha\beta\gamma}$ (in the model (3)) yields Eq.(5) where L_1 is now defined, according to Eq.(1), as the part of the integrand of the action (17) coupled to the measure Φ . The appearance of the integration constant sM^4 in Eq.(5) spontaneously breaks the global scale invariance (18).

Except for the A_μ equation, all other equations of motion resulting from (17) in the first order formalism contain terms proportional to $\partial_\mu \zeta$ that makes the space-time non-Riemannian and equations of motion - non canonical. However, with the new set of variables (ϕ and A_μ remain unchanged)

$$\tilde{e}_{a\mu} = e^{\frac{1}{2}\alpha\phi/M_p}(\zeta + b)^{1/2}e_{a\mu}, \quad \tilde{g}_{\mu\nu} = e^{\alpha\phi/M_p}(\zeta + b)g_{\mu\nu}, \quad \Psi'_i = e^{-\frac{1}{4}\alpha\phi/M_p} \frac{(\zeta + k)^{1/2}}{(\zeta + b)^{3/4}} \Psi_i, \quad i = N, E \quad (19)$$

which we call the Einstein frame, the spin-connections become those of the Einstein-Cartan space-time. Since $\tilde{e}_{a\mu}$, $\tilde{g}_{\mu\nu}$, N' and E' are invariant under the scale transformations (18), spontaneous breaking of the scale symmetry (by means of Eq.(5)) is reduced in the new variables to the *spontaneous breaking of the shift symmetry* [21]

$$\phi \rightarrow \phi + \text{const.} \quad (20)$$

Notice that the Goldstone theorem generically is not applicable in this theory (see the second reference in Ref. [4]). The reason is the following. In fact, the scale symmetry (18) leads to a conserved dilatation current j^μ . However, for example in the spatially flat FRW universe the spatial components of the current j^i behave as $j^i \propto M^4 x^i$ as $|x^i| \rightarrow \infty$. Due to this anomalous behavior at infinity, there is a flux of the current leaking to infinity, which causes the non conservation of the dilatation charge. The absence of the latter implies that one of the conditions necessary for the Goldstone theorem is missing. The non conservation of the dilatation charge is similar to the well known effect of instantons in QCD where singular

behavior in the spatial infinity leads to the absence of the Goldstone boson associated to the $U(1)$ symmetry.

V. TMT STRUCTURE, COSMOLOGICAL CONSTANT PROBLEM AND AVOIDANCE OF THE WEINBERG NO-GO THEOREM

One can show [8], [11] that in the absence of massive fermions the constraint determines ζ as the function of ϕ alone:

$$\zeta = \zeta_0(\phi) \equiv b - \frac{2V_2}{V_1 + M^4 e^{-2\alpha\phi/M_p}}, \quad (21)$$

Note that the electromagnetic field does not enter in the constraint and therefore the presence of the electromagnetic field does not affect the value of ζ_0 .

The effective potential of the scalar field ϕ produced in the Einstein frame reads

$$V_{eff}^{(0)}(\phi) \equiv V_{eff}(\phi; \zeta_0)|_{\psi', \psi''=0} = \frac{[V_1 + M^4 e^{-2\alpha\phi/M_p}]^2}{4[b(V_1 + M^4 e^{-2\alpha\phi/M_p}) - V_2]} \quad (22)$$

and the ϕ -equation has the standard form

$$(-\tilde{g})^{-1/2} \partial_\mu (\sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_\nu \phi) + V_{eff}^{(0)'}(\phi) = 0, \quad (23)$$

where prime sets derivative with respect to ϕ .

The mechanism of the appearance of the effective potential (22) is very interesting and exhibits the main features of our TMT model⁵. In fact, the reasons of the transformation of the prepotentials $V_1 e^{2\alpha\phi/M_p}$ and $V_2 e^{2\alpha\phi/M_p}$, coming in the original action (17), into the effective potential (22) are the following:

⁵The particular case of this model with $b = 0$ and $V_2 < 0$ was studied in Ref. [4]. The application of the TMT model with explicitly broken global scale symmetry to the quintessential inflation scenario was discussed in Ref. [5]. A particular case of the model (17) without explicit potentials, i.e. $V_1 = V_2 = 0$, has been studied in Ref. [8].

- a) Transformation to the Einstein frame;
- b) Spontaneous breakdown of the global scale symmetry which in the Einstein frame is reduced to the spontaneously broken shift symmetry (20);
- c) The constraint which in the absence of fermions case takes the form (21).

It turns out that the gravitational equations in the absence of fermions case become the standard Einstein equations of the model where the electromagnetic field and the minimally coupled scalar field ϕ with the potential $V_{eff}^{(0)}(\phi)$ are the sources of gravity.

The most remarkable feature of the effective potential (22) is that it is proportional to the square of $V_1 + M^4 e^{-2\alpha\phi/M_p}$. Due to this, as $V_1 < 0$ and $bV_1 - V_2 > 0$, *the effective potential has a minimum where it equals zero automatically*, without any further tuning of the parameters V_1 and V_2 . This occurs in the process of evolution of the field ϕ at the value of $\phi = \phi_0$ where

$$V_1 + M^4 e^{-2\alpha\phi_0/M_p} = 0. \quad (24)$$

This means that the universe evolves into the state with zero cosmological constant without tuning parameters of the model.

If such type of the structure for the scalar field potential in a usual (non TMT) model would be chosen "by hand" it would be a sort of fine tuning. But in our TMT model it is not the starting point, *it is rather a result* obtained in the Einstein frame of TMT models with spontaneously broken global scale symmetry including the shift symmetry $\phi \rightarrow \phi + const.$ The same effect can be obtained in more general models including for example the Higgs field as well [11]. Note that the assumption of scale invariance is not necessary for the effect of appearance of the perfect square in the effective potential in the Einstein frame and therefore for the described mechanism of disappearance of the cosmological constant, see Refs. [2]- [4].

On the first glance this effect contradicts the no-go Weinberg theorem [22] which states that there cannot exist a field theory model where the cosmological constant is zero without fine tuning. Recall that one of the basic assumptions of this no-go theorem is that all fields

in the vacuum must be constant. However, this is not the case in TMT. In fact, in the vacuum determined by Eq.(24) the scalar field $\zeta \equiv \frac{\Phi}{\sqrt{-g}}$ is non zero, see Eq.(21). The latter is possible only if all the φ_a ($a = 1, 2, 3, 4$) fields (in the definition of Φ by means of Eq.(2)) or the 3-index potential $A_{\alpha\beta\gamma}$ (when using the definition of Φ by means of Eq.(3)) have non vanishing space-time gradients. Moreover, exactly in the vacuum $\phi = \phi_0$ the scalar field ζ has a singularity. However, in the conformal Einstein frame all physical quantities are well defined and this singularity manifests itself only in the vanishing of the vacuum energy density. We conclude therefore that the Weinberg theorem [22] is not applicable in the context of the TMT models studied here. In fact, the possibility of such type of situation was suspected by S. Weinberg in the footnote 8 of his review [22] where he pointed out that when using a 3-index potential with non constant vacuum expectation value, his theorem does not apply.

When considering more realistic models, the pre-potentials V_1 and V_2 turn into functions of the Higgs field. If V_1 and V_2 include for example both Higgs mass terms and quartic Higgs self-interactions, then there could be vacua with zero cosmological constant disconnected from each other (again without fine tuning). This is an explicit realization of the "Multiple Point Principle" proposal [23] which is based on the idea that if there is a mechanism that sets a certain state to have a zero cosmological constant then the same mechanism may act also in other field configurations with the same result.

VI. PHENOMENOLOGICAL OUTPUT OF TMT

Here we list some of the most interesting results of TMT:

1. Restoration of GR as local fermion energy density is much larger than the scalar dark energy in the space-time region occupied by the fermion [8]- [10]:
 - a) restoration of the Einstein equations;
 - b) decoupling of the scalar field ϕ from fermionic matter;
 - c) some ideas concerning the nature of three fermion generations are also discussed.

2. As local fermion energy density has the order of magnitude close to that of the scalar dark energy, the fermion may be in an exotic state [10]: gas of nonrelativistic neutrinos in such a state behaves as dark energy.
3. Power law inflation of the early universe, driving by the scalar sector (inflaton ϕ and the Higgs field v), consists of two stages [11]. In the second stage, starting about 50 e-folding before the end of inflation, v performs quickly damping oscillations around $v = 0$.
4. Soon after the end of inflation, a transition to the gauge symmetry broken phase, which at the same time is a true vacuum state with zero energy density, is realized [11]. This transition is accompanied with the Higgs field oscillations that should serve as an effective mechanism of reheating.
5. Zero vacuum energy and spontaneous breakdown of the gauge symmetry occur without any fine tuning and without tachyonic mass term in the action [11].
6. Other scenario is that of early inflation driven by R^2 term, followed by transition to slowly accelerated phase [6].
7. For the late time universe a possibility of superaccelerated expansion of the universe is shown without phantom field in the action [11].
8. Smallness of the present cosmological constant may be obtained by a see-saw mechanism [10], [11].

VII. PROBLEMS OF TMT QUANTIZATION. PRELIMINARY DISCUSSION

So far we have discussed effects only in classical two measures field theory. However quantization of TMT as well as influence of quantum effects on the processes explored might have a crucial role. We summarize here some ideas and speculations which gives us a hope that quantum effects can keep and even strengthen the main results displayed in classical TMT.

- 1) Recall first two fundamental facts of TMT as a classical field theory: (a) The measure degrees of freedom appear in the equations of motion only via the scalar ζ ; (b) The scalar ζ

is determined by the constraint which is nothing but a consistency condition of the equations of motion. Therefore the constraint plays a key role in TMT. Note however that if we were ignore the gravity from the very beginning in the action (17) then instead of the constraint we would obtain a very different equation, i.e. in such a case we would deal with a different theory. This notion shows that the gravity and matter intertwined in TMT in a much more complicated manner than in GR. Hence introducing the new measure of integration Φ we have to expect that the quantization of TMT may be a complicated enough problem. Nevertheless we would like here to point out that in the light of the recently proposed idea of Ref. [24], the incorporation of four scalar fields φ_a together with the scalar density Φ , Eq.(2), (which in our case are the measure fields and the new measure of integration respectively), is a possible way to define local observables in the local quantum field theory approach to quantum gravity. We regard this result as an indication that the effective gravity + matter field theory has to contain the new measure of integration Φ as it is in TMT. In fact in our motivation of TMT from the point of view of the brane world scenario, Eq.(8), the measure fields φ_a have a meaning of the coordinates of the embedding 3-brane. This supports both the idea of the measure fields as physical coordinates [24] and of the structure of TMT expressed by Eq.(1).

2) The assumption that the measure fields φ_a (or $A_{\alpha\beta\gamma}$) appear in the action (1) *only* via the measure of integration Φ , has a key role in the TMT results and in particular for the resolution of the old cosmological constant problem. In principle one can think of breakdown of such a structure by quantum corrections. However, fortunately there exists an infinite dimensional symmetry (16) (and its analog in the case (3)) which, as we hope, is able to protect the postulated structure of the action from a deformation caused by quantum corrections or at least to suppress such a quantum anomaly in significant degree. Therefore one can hope that the proposed resolution of the old cosmological constant problem holds in the quantized TMT as well.

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